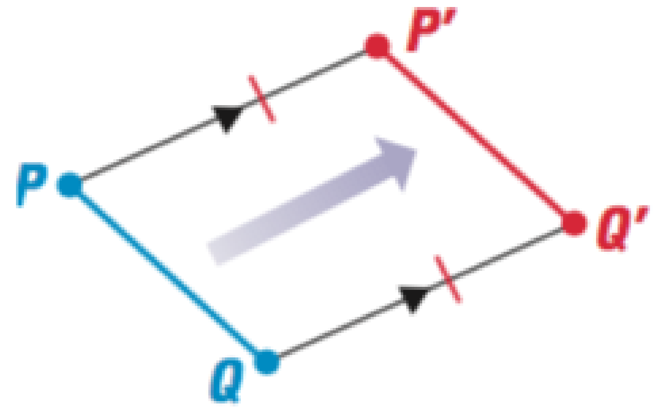
Chapter 7
Transformations

Section 4
Translations and Vectors

GOAL 1: Using Properties of Translations

A **translation** is a transformation that maps every two points P and Q in the plane to points P' and Q', so that the following properties are true:



1. $PP' = QQ'$
2. $\overline{PP'} \parallel \overline{QQ'}$, or $\overline{PP'}$ and $\overline{QQ'}$ are collinear.

THEOREM

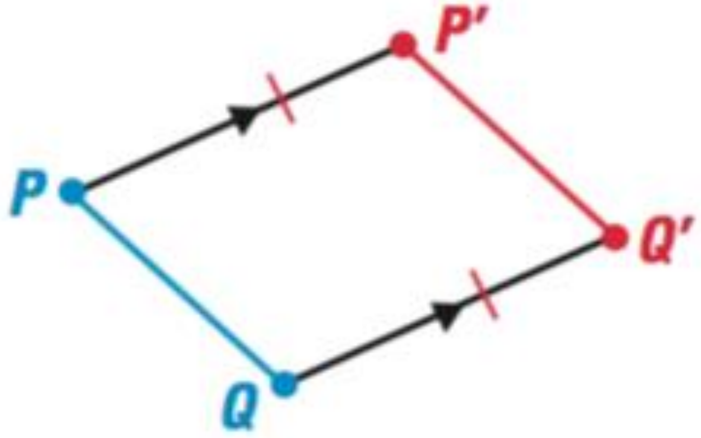
THEOREM 7.4 ***Translation Theorem***

A translation is an isometry.

Theorem 7.4 can be proven as follows.

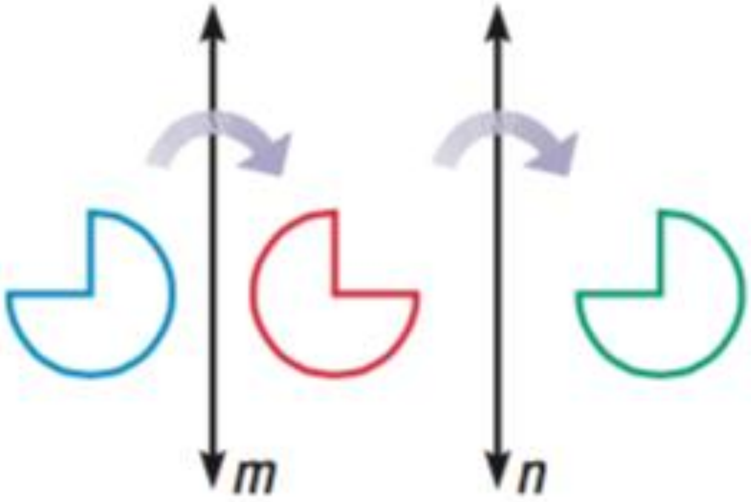
GIVEN ▶ $PP' = QQ'$, $\overline{PP'} \parallel \overline{QQ'}$

PROVE ▶ $PQ = P'Q'$



Paragraph Proof The quadrilateral $PP'Q'Q$ has a pair of opposite sides that are congruent and parallel, which implies $PP'Q'Q$ is a parallelogram. From this you can conclude $PQ = P'Q'$. (Exercise 43 asks for a coordinate proof of Theorem 7.4, which covers the case where $\overline{PQ}$ and $\overline{P'Q'}$ are collinear.)

You can find the image of a translation by gliding a figure in the plane. Another way to find the image of a translation is to complete one reflection after another in two parallel lines, as shown. The properties of this type of translation are stated below.

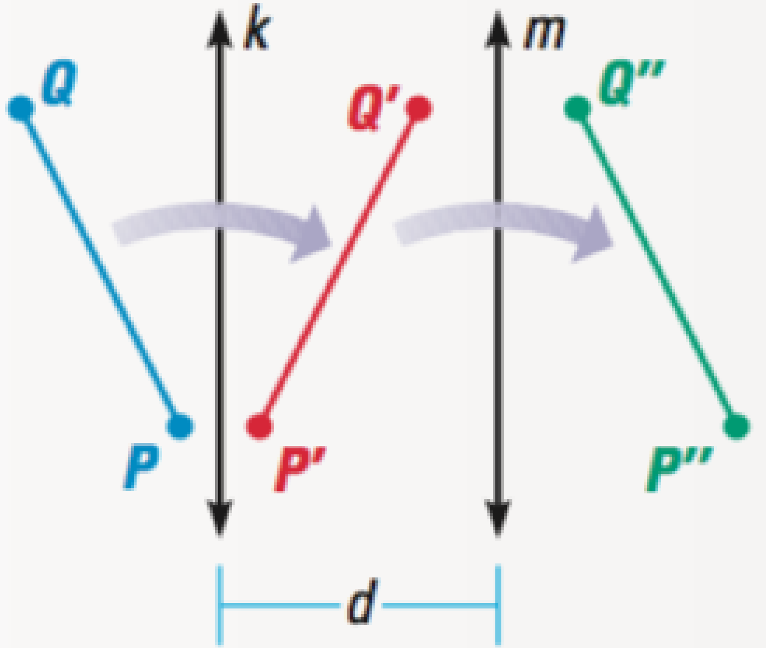


THEOREM

THEOREM 7.5

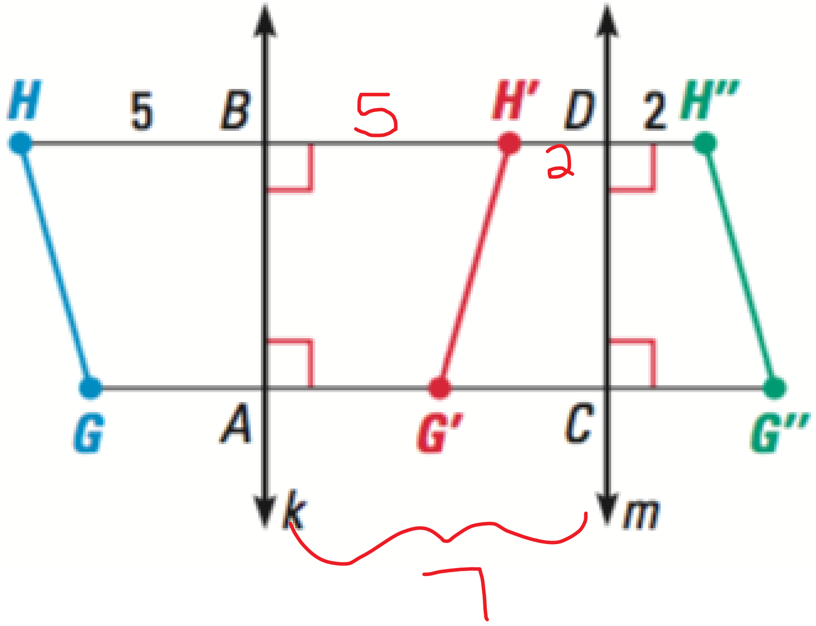
If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is a translation. If P'' is the image of P, then the following is true:

1. $\overleftrightarrow{PP''}$ is perpendicular to k and m.
2. $PP'' = 2d$, where d is the distance between k and m.



Example 1: Using Theorem 7.5

In the diagram, a reflection in line k maps GH to G'H', a reflection in line m maps G'H' to G"H", k||m, HB = 5, and DH" = 2.



a) Name some congruent segments.

HG = H'G' = H"G"; BD = AC; HB = H'B;

GA = G'A; H'D = H"D; G'C = G"C

a) Does AC = BD? Explain.

Yes, b/c k || m → distance is same between the lines at all points

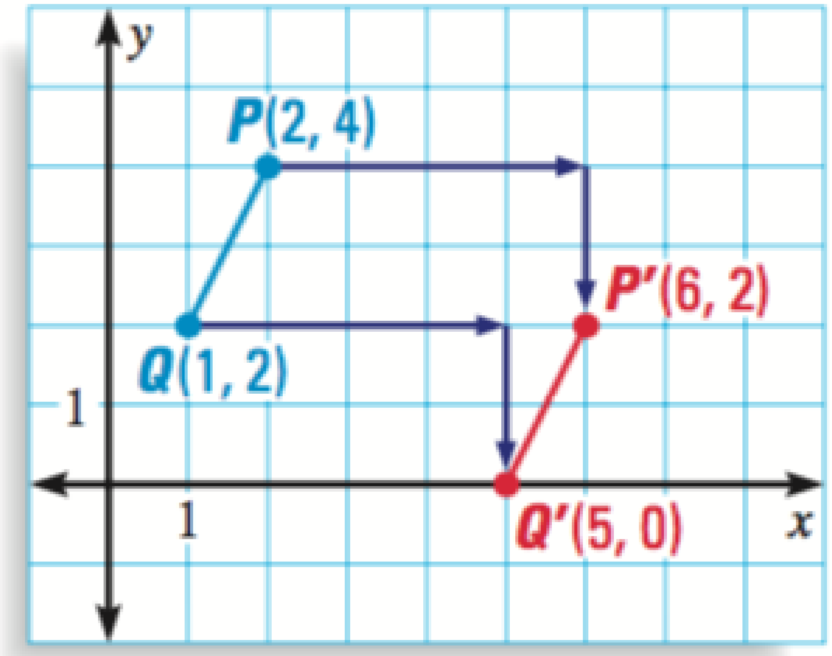
a) What is the length of GG"?

14 (use 7.5 → 7 x 2 OR HH" = 14 → GG" = 14 as well)

Translations in a coordinate plane can be described by the following coordinate notation:

$$(x, y) \rightarrow (x \pm a, y \pm b)$$

where a and b are constants. Each point shifts a units horizontally and b units vertically. For instance, in the coordinate plane at the right, the translation $(x, y) \rightarrow (x + 4, y - 2)$ shifts each point 4 units to the right and 2 units down.

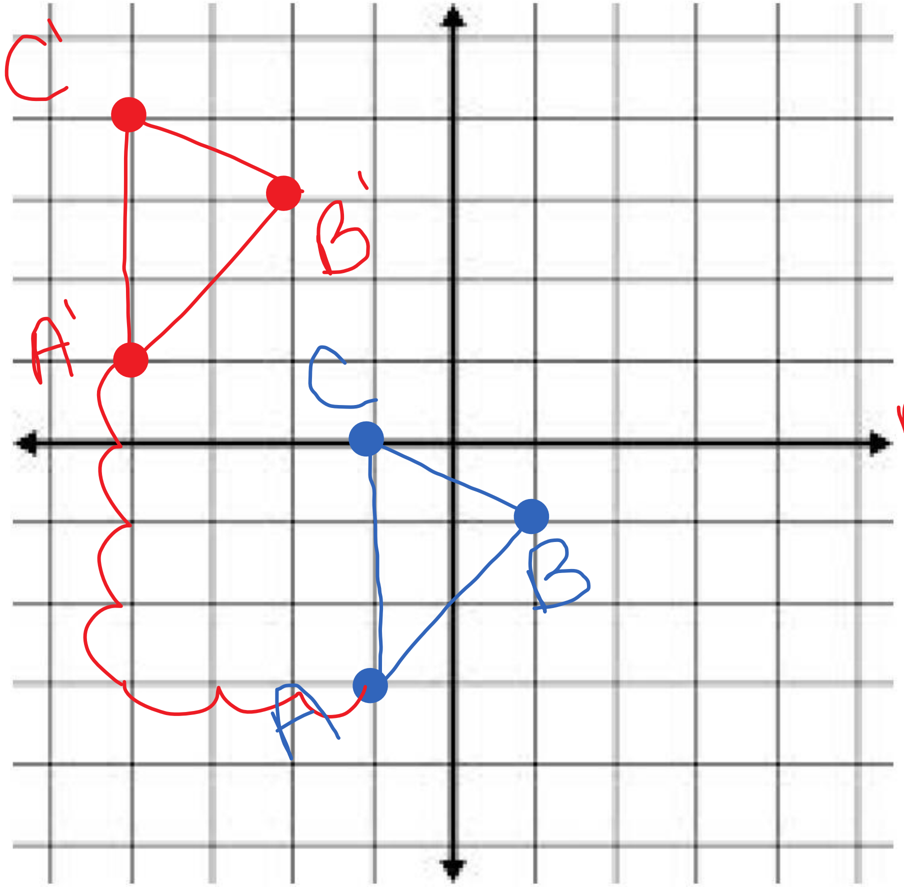


right/up +

left/down −

Example 2: Translations in a Coordinate Plane

Sketch a triangle with vertices A(-1, -3), B(1, -1) and C(-1, 0). Then sketch the image of the triangle after the translation $(x, y) \rightarrow (\underbrace{x - 3}_{\text{left 3}}, \underbrace{y + 4}_{\text{up 4}})$.



A'(-4, 1)

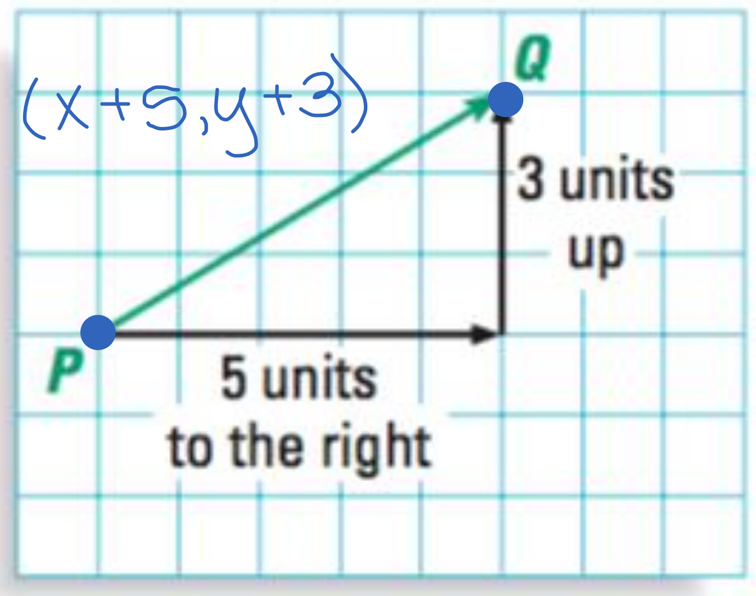
B'(-2, 3)

C'(-4, 4)

GOAL 2: Translations Using Vectors

Another way to describe a translation is by using a vector. A **vector** is a quantity that has both direction and *magnitude*, or size, and is represented by an arrow drawn between two points.

The diagram shows a vector. The **initial point**, or starting point, of the vector is P and the **terminal point**, or ending point, is Q. The vector is named $\overrightarrow{PQ}$, which is read as "vector PQ." The *horizontal component* of $\overrightarrow{PQ}$ is 5 and the *vertical component* is 3.

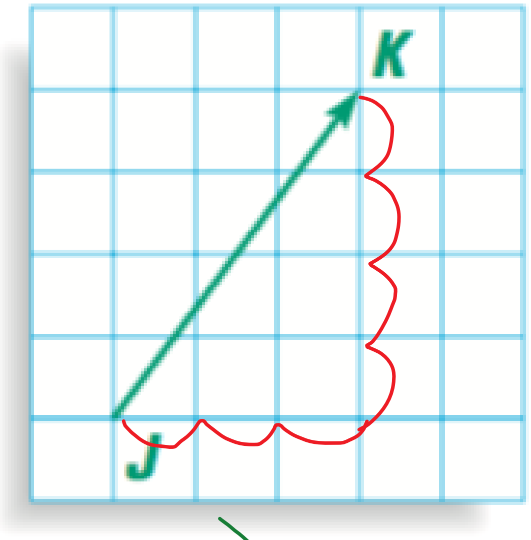


The **component form** of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{PQ}$ is $\langle 5, 3 \rangle$.

Example 3: Identifying Vector Components

In the diagram, name each vector and write its component form.

a.

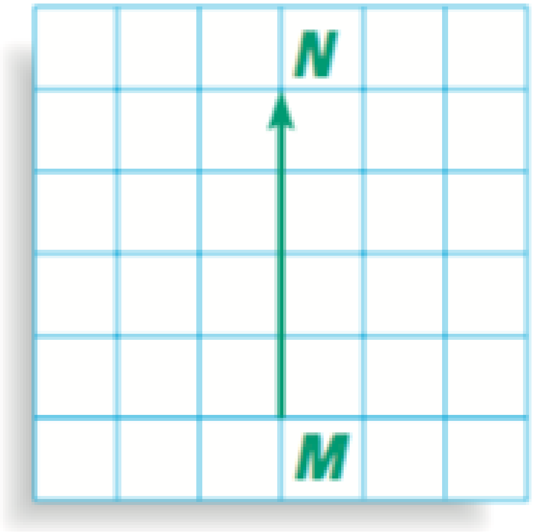


$\overrightarrow{JK}$

R3, U4

$\langle 3, 4 \rangle$

b.

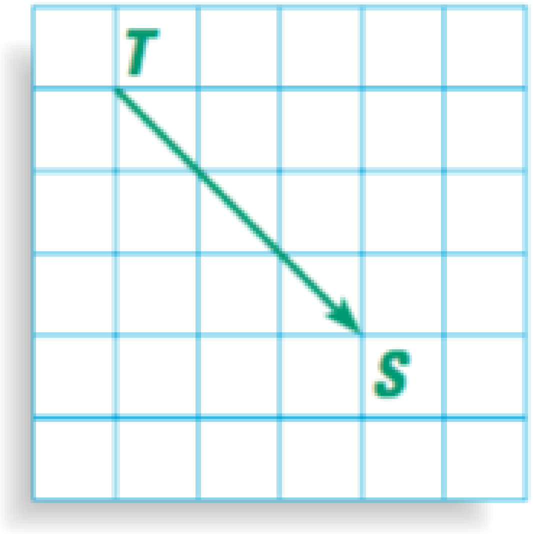


$\overrightarrow{MN}$

R0, U4

$\langle 0, 4 \rangle$

c.



$\overrightarrow{TS}$

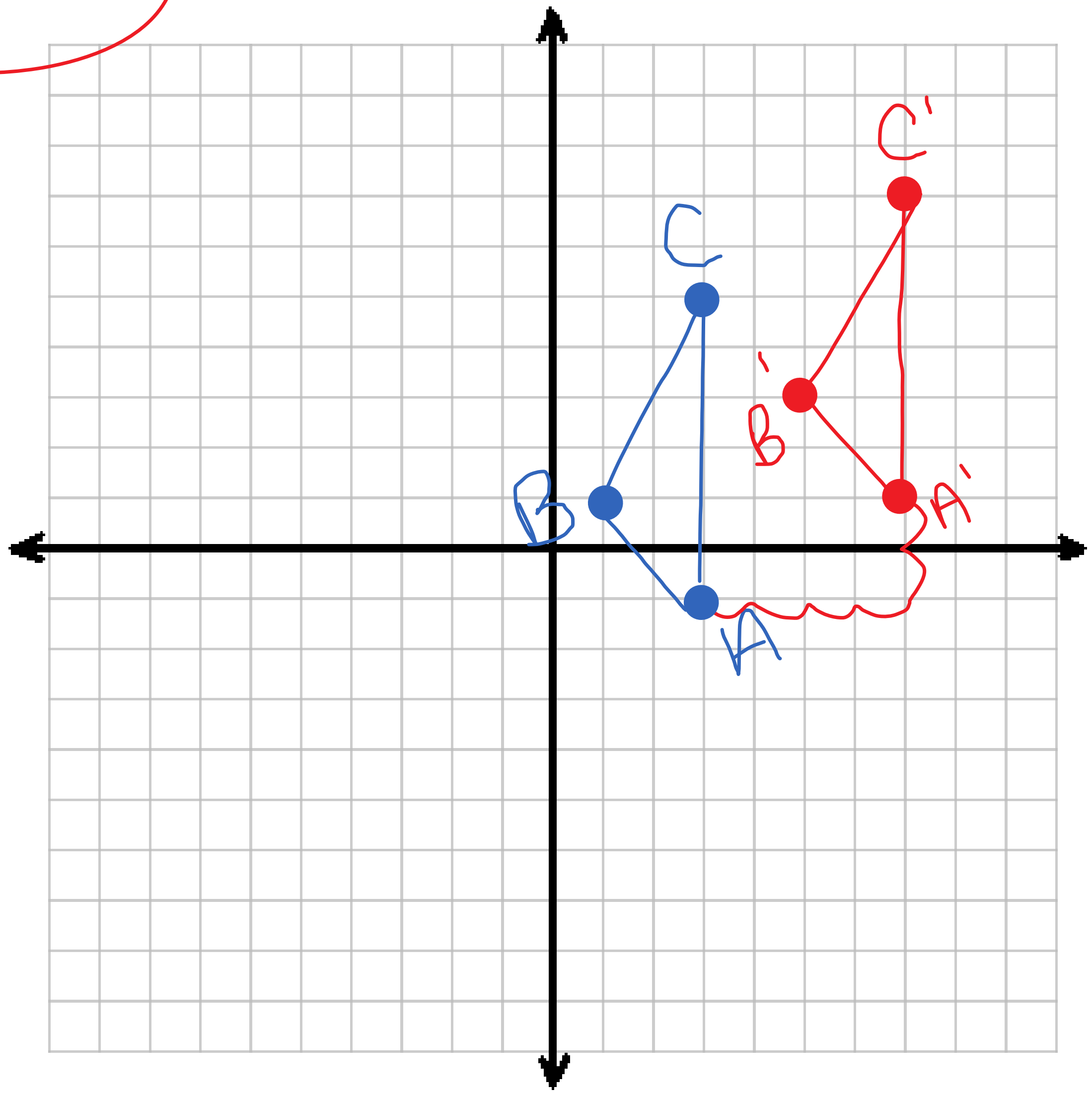
R3, D3

$\langle 3, -3 \rangle$

Example 4: Translations Using Vectors

The component form of $\overrightarrow{GH}$ is $\langle 4, 2 \rangle$. Use $\overrightarrow{GH}$ to translate the triangle whose vertices are $A(3, -1)$, $B(1, 1)$, and $C(3, 5)$.

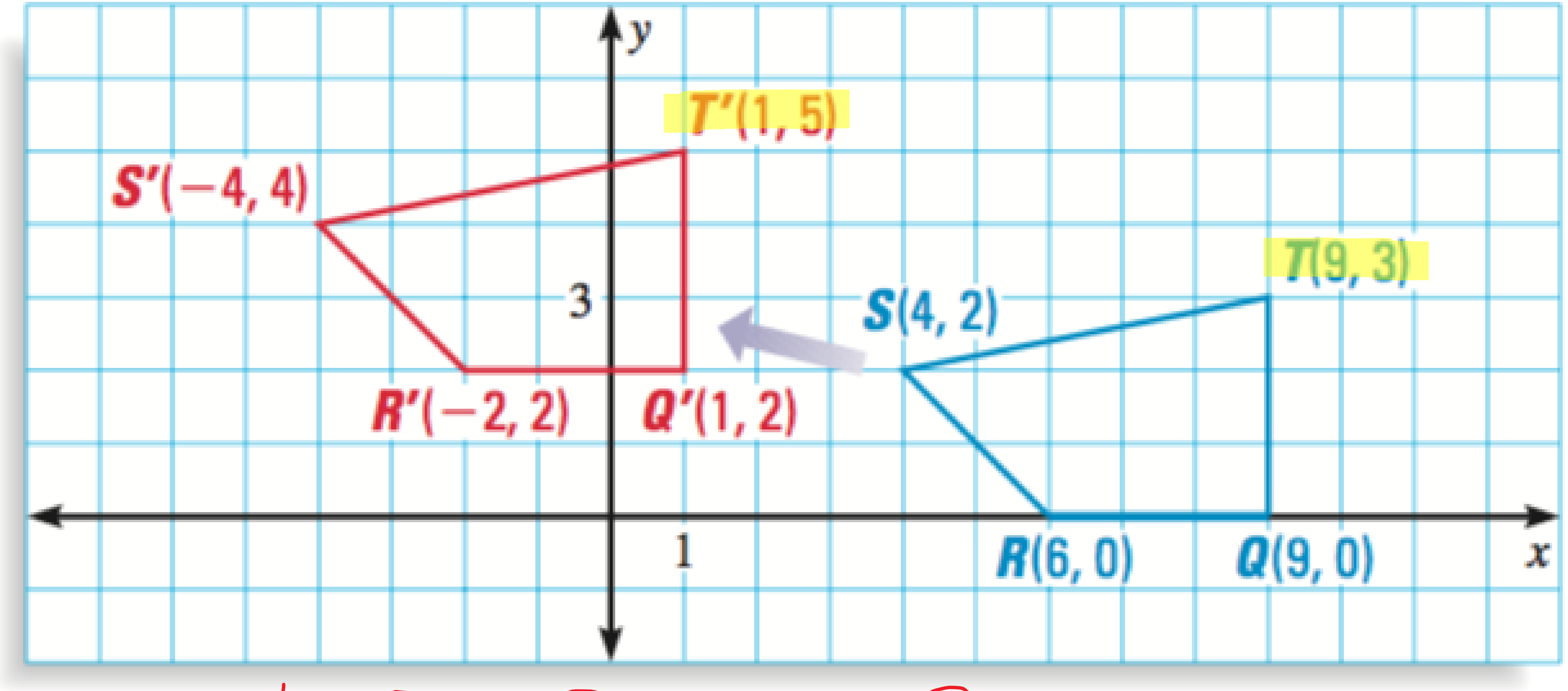
4 right
2 up



A'(7, 1)
B'(5, 3)
C'(7, 7)

Example 5: Finding Vectors

In the diagram, $QRST$ maps onto $Q'R'S'T'$ by a translation. Write the component form of the vector that can be used to describe the translation.



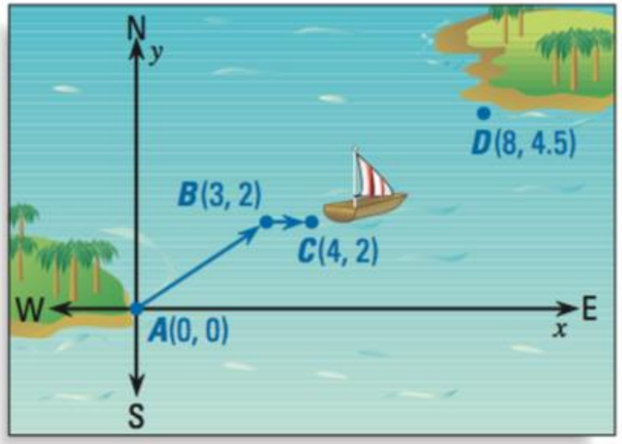
left 8, up 2

CN: $(x, y) \rightarrow (x-8, y+2)$

CF: $\langle -8, 2 \rangle$

Example 6: Using Vectors

Navigation: A boat travels a straight path between two islands, A and D. When the boat is 3 miles east and 2 miles north of its starting point, it encounters a storm at point B. The storm pushed the boat off course to point C, as shown.



Write the component forms of the two vectors shown in the diagram.

$\overrightarrow{AB}$ $\langle 3, 2 \rangle$

$\overrightarrow{BC}$ $\langle 1, 0 \rangle$

The final destination is 8 miles east and 4.5 miles north of the starting point. Write the component form of the vector that describes the path the boat can follow to arrive at its destination.

$\overrightarrow{CD}$ $\langle 4, 2.5 \rangle$

EXIT SLIP